Function Spaces and Distribution Theory

Course Format
Foundation Module – Dr Antonio Esposito

Duration
8 hours

Overview
This course will be an introduction, in the spirit of a user's guide, to modern techniques in Analysis, which are central to the theoretical and numerical treatment of random systems.

Learning Outcomes
Students will learn basic techniques and results about Lebesgue and Sobolev spaces, distributions and weak derivatives, embedding and trace theorems, and weak convergence.

Synopsis

- Revision of differential calculus: Fréchet differentiability.

- Revision of relevant definitions and statements from Lebesgue integration theory: convergence theorems, completeness, separability and duality.

- Weak and weak* convergence in Lebesgue spaces: oscillation and concentration. Equi-integrability and Vitali's convergence theorem. Examples. Statement of Banach–Alaouglu Theorem.

- Mollifiers and density of smooth functions in L^p, $1 \leq p < \infty$.

- Distributions and distributional derivatives. Operations on distributions. Examples.

- Sobolev Spaces: mollifications and weak derivatives, separability and completeness. Poincaré and Sobolev Inequalities. Extensions and traces. Embedding Theorems and compactness.

Prerequisites
Basic Functional Analysis and Lebesgue Integration.

Core Reading

Walter Rudin, Functional Analysis, McGraw-Hill, 2nd edition (1 Jan. 1991)

H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Universitext, Springer, New York, 2011

Sobolev Spaces, by R.A. Adams and J.J.F. Fournier

L.C. Evans and R.F. Gariepy, Measure Theory and Fine Properties of Functions, CRC Press, Boca Raton, FL, 1992

P. D. Lax, Functional Analysis, John Wiley & Sons, Inc., New York, 2020

E.H. Lieb and M. Loss, Analysis, 2nd Edition, Graduate Studies in Mathematics, American Mathematical Society, 2001

E.M. Stein and R. Shakarchi, Real Analysis. Measure Theory, Integration and Hilbert Spaces, Princeton Lectures in Analysis, III. Princeton University Press, Princeton, NJ, 2005