Advanced Topics in Stochastic Analysis
CDT in Maths of Random Systems

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Good things come in threes, they say, and this course is no different. Indeed, we will focus on the following three topics:

1. Stochastic integration for processes with jumps
2. Weak and u.c.p. convergence theory for stochastic integrals
3. Interacting particle systems and McKean–Vlasov problems

Description of the course

In the first part of the course, we will give a self-contained development of the theory of stochastic integration for processes with jumps, and we will briefly discuss stochastic differential equations for such processes. Our approach is inspired by that of Philip Protter's book on the subject, which can serve as useful background reading, but self-contained notes will be provided. Even for continuous processes, this approach will shed a rather different light on the theory from your foundations course on stochastic analysis, focusing on what it means to be a 'good integrator', as opposed to taking the notion of a (continuous or discontinuous) semimartingale for granted. Given the exposure to key ideas in the foundations course, we will move at a brisk pace in certain places; however, the main results will be carefully proved and motivated.

In the second part of the course, we will first cover the classical theory of Skorokhod topologies on the space of càdlàg processes, and then we will have a detailed look at weak convergence for probability measures on this space (including an introduction to Wasserstein distances). Based on this, we proceed to develop a weak convergence theory for stochastic integrals with jumps (building on the first part of the course), which leads to stability results of great theoretical and practical interest.

In the final part of the course, we will discuss law of large numbers type results for large systems of interacting stochastic differential equations. In particular, we will see how the results from the second part of the course can be used to establish the weak convergence of such systems, leading us to the study of McKean–Vlasov problems. We will give a overview introduction to this area, and, as time allows, explore some particular interesting examples with jumps, as well as discuss connections to partial differential equations, both stochastic and deterministic.

Lecture notes and preparation

The classes will be supplemented by a tailored set of lecture notes. The classes will only cover the most important ideas from these notes, so students are expected to familiarise themselves with parts of the notes not covered directly in class. In terms of preparation, the course will assume a good level of fluency with the topics covered in the introductory courses on *Function Spaces and Distributions* and the *Foundations of Stochastic Analysis*. Other than this, the course is self-contained. Concerning the first part of the course, two good books are (i) '*Stochastic Integration and Differential Equations*' by Philip Protter, and (ii) '*Levy Processes and Stochastic Calculus*' by David Applebaum.