Advanced Topics in Stochastic Analysis and Control
CDT in Mathematics of Random Systems

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This course will focus on:

- McKean–Vlasov forward-backward stochastic differential equations (SDEs), interacting particle systems, weak convergence of probability measures and Wasserstein metrics;
- Theory and application of mean-field control problems;
- Theory of stochastic integration for processes with jumps, theory of SDEs with jumps.

1 Description

In the main part of the course, we aim to develop a framework for the study of mean-field control problems. After a brief recap of aspects from the classical stochastic control theory [4], we continue with a review of the basics of weak convergence of probability measures, along with some discussion of Wasserstein metrics, weak convergence and concepts related to variational calculus on the Wasserstein space. Before turning to mean-field control problems, we devote some time to explore interacting particle systems, the notion of propagation of chaos [2], and questions concerning the well-posedness of McKean–Vlasov forward-backward SDEs [1]. We will then consider the control of McKean–Vlasov dynamics whose coefficients have mean-field interactions in the state and control and establish a Pontryagin stochastic maximum principle, both in necessary and in sufficient form [1].

Further, we will study some aspects of the theory of stochastic integration for processes with jumps, as presented in Protter [3], i.e., we define a semimartingale as a stochastic process which is a good integrator on an elementary class of processes (rather than as one that can be written as the sum of a local martingale and an adapted process with paths of finite variation on compacts). If time permits, we will additionally investigate well-posedness of SDEs subject to jumps.

References

[1] R. Carmona and F. Delarue, *Probabilistic Theory of Mean Field Games with Applications I*, Vol. 84 of Probability Theory and Stochastic Modelling, Springer International Publishing, 1st ed., (2018)

[2] A. S. Sznitman, *Topics in Propagation of Chaos*, Ecole d'été de probabilités de Saint-Flour XIX - 1989, Vol. 1464 of Lecture notes in Mathematics, Springer-Verlag, (1991)

[3] P. E. Protter, *Stochastic integration and differential equations*, Vol. 21 of Stochastic modelling and applied probability, Springer–Verlag Berlin Heidelberg, 2nd ed., (2005)

[4] J. Yong and X. Y. Zhou, *Stochastic Controls: Hamiltonian Systems and HJB Equations*, Springer-Verlag, New York, (1999)